# Housing Fairness Evaluation – Based on Housing Relative Deprivation Index

By Xu Yuejin[1] and Liu Hongyu[2]
1: Room 430, Institute of Real Estate Studies, Building 10#, Tsinghua University, Beijing, China
Email: xuyuejin8@126.com
2: Room 433, Institute of Real Estate Studies, Building 10#, Tsinghua University, Beijing, China
Email: liuhy@tsinghua.edu.cn

**Abstract:** Fairness is the lofty value which mankind pursues. As an important part of social fairness, housing fairness holds more and more attention in recent years. This paper introduces relative deprivation to quantitatively measure housing fairness. Housing relative deprivation indexes (HRD) are constructed based on two distributions, the current residential level and the reachable residential level, separately. Central China presents the highest degree of housing fairness while Western China has the greatest decreasing level of housing fairness from 2000 to 2010 from HRD based on the current residential level. The index based on the reachable residential level, considering the influence of household income and housing price, is used to evaluate the impact of housing price control policy and housing security policy on housing fairness. For housing price control policy, the index shows that it cannot improve housing fairness effectively by slightly lowering (10%~20%) the current (year 2010) housing price. And the discount of housing price has to be at least up to 30% in order to pull the housing relative deprivation down. Then, efficiencies of two policies are compared under housing security policy. The result indicates that the efficiency of low-cost social housing policy in improving housing fairness is higher than that of the owner-occupied housing policy with same government input.

**Key words:** relative deprivation, housing fairness, housing price control, housing security

## 1. Introduction

Fairness is the first virtue of social institutions, as truth is of systems of thought [1]. And it is clearly pointed out in, former China President, Hu Jintao's report at 18th Party Congress that we should ensure equal rights, equal opportunities and fair rules for all by developing institutions, establishing system and fostering a fair social environment. As an important part of social fairness, housing fairness plays a vital role.

The efficiency of China's real estate market increases constantly, and people's living standard continues to improve with the deepening of the housing system reform in the last 30 years. The per capita floor area of urban residents has reached 31.93 $m^2$ in 2010 from 23.29 $m^2$ in 2000 [2]. However, housing fairness does not come along with economic development. Problems of housing fairness, such as uneven housing distribution, low housing affordability of urban residents, as well as the bad housing environment of poor families, become serious.

There is an urgent need for analysis of housing fairness. However, there have been only a few quantitative studies on housing fairness so far. It is mainly because of the difficulty to construct appropriate indicator of fairness perception. Luckily, the study of relative deprivation provides an

innovative approach for measuring fairness.

Relative deprivation is a type of deprivation having a different concepts with absolute deprivation. Absolute deprivation is the poverty level that fall below minimum requirements that necessary to live a decent life, afford minimum basic needs such as shelter, food and healthcare. And relative deprivation is the lack of resources to sustain the diet, lifestyle, activities and amenities that an individual or group are accustomed to or that are widely encouraged or approved in the society to which they belong. Absolute deprivation is related to poverty, while relative deprivation is more about fairness perception.

The perception of fairness is actually derived from comparison, which is described vividly by *Karl Marx* (1902) [3]. As the saying goes, inequality rather than want is the cause of trouble.

*A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirements for a residence. But let there arise next to the little house a palace, and the little house shrinks into a hut. The little house now makes it clear that its inmate has no social position at all to maintain, or but a very insignificant one; and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

— *Karl Marx*

Based on fairness perception and relative deprivation, the ***housing relative deprivation index*** (hereinafter referred to as HRD) is constructed in this paper for the evaluation of housing fairness in China. The indexes employed in this paper are calculated based on two different distribution of residential level, one is the current residential level, and the other is the reachable residential level. The index based on the current residential level, which reflects fairness degree of housing allocation at a certain time and space, could provide supportive information for policy makers. The other index based on the reachable residential level, which further considers the influence of household income and housing price, could be used to evaluate the impact of different policies on housing fairness.

The rest of this paper goes as follows. Housing relative deprivation and related models are introduced in section 2. When the distribution of residential level is given, HRD could be calculated accordingly. Based on the distribution of current residential level, HRD of China and various provinces are calculated and analyzed in section 3. Further, the model of housing relative deprivation based on the reachable residential level is showed in section 4. Then the index showed in section 4 is used to evaluate the impact of housing price control policy and housing security policy in section 5. Conclusion would be given in the last.

# 2. Housing Relative Deprivation Index

## 2.1. Previous model

From an economic standpoint, *Runciman* (1966) [4] defines relative deprivation as follows: a person is relatively deprived of $\boldsymbol{Z}$ when (1) he does not have $\boldsymbol{Z}$, (2) he sees some other person or persons, which may include himself at some previous or expected time, as having $\boldsymbol{Z}$, (3) he wants $\boldsymbol{Z}$, and (4) he sees it as feasible that he should have $\boldsymbol{Z}$. This definition has a significant impact on the quantitative model of relative deprivation.

Some different quantitative models have been constructed to calculate relative deprivation, among which the model constructed by *Hey* and *Lambert* (1980) had probably the greatest impact

in this field [5-7]. ***Referring to their model, we define the housing relative deprivation of target household to be the difference of residential levels (measured by per capita floor area) between target and reference households in the distribution.*** Specifically, when the per capita floor area of the reference household is ***z***, the household with the per capita floor area ***y*** (hereinafter referred to as *household* ***y***) would have the housing relative deprivation $D(y,z)$,

$$\mathrm{D}(\mathrm{y},\mathrm{z}) = \begin{cases} z-y & y<z \\ 0 & y \geq z \end{cases} \qquad (1)$$

Then, based on (1), model of housing relative deprivation within and between groups is constructed referring to *Elena's* approach (2007) [8]. Specifically, household ***y*** and ***z*** are in group ***Y*** and ***Z*** separately, where $F_Y(\cdot)$ and $F_Z(\cdot)$ are the distribution functions. Then the deprivation of household ***y*** with reference to household ***z***, $D_{YZ}(y,z)$, is given by,

$$\mathrm{D}_{YZ}(\mathrm{y},\mathrm{z}) = \begin{cases} z-y & y<z \\ 0 & y \geq z \end{cases} \qquad (2)$$

When comparing all the households in group ***Z*** with household ***y***, the deprivation of household ***y*** with reference to group ***Z*** is,

$$\mathrm{D}_{YZ}(\mathrm{y}) = \int_0^\infty \mathrm{D}_{YZ}(\mathrm{y},\mathrm{z})\, dF_Z(\mathrm{z}) = \int_y^\infty z-y\, dF_Z(\mathrm{z}) \qquad (3)$$

Further, the average housing relative deprivation of group ***Y*** with reference to group ***Z***, denoted by $D_{YZ}$, is given by,

$$\mathrm{D}_{YZ} = E\big(\mathrm{D}_{YZ}(\mathrm{y})\big) = \int_0^\infty \mathrm{D}_{YZ}(\mathrm{y})\, dF_Y(\mathrm{y}) = \int_0^\infty \int_y^\infty z-y dF_Z(\mathrm{z}) dF_Y(\mathrm{y}) \qquad (4)$$

In particular, when ***Y*** equals to ***Z***, housing relative deprivation $D_{YZ}$ becomes the average relative deprivation of group ***Y*** with reference to itself, hence $D_Y$ is,

$$\mathrm{D}_Y = E\big(\mathrm{D}_Y(\mathrm{y})\big) = \int_0^\infty \mathrm{D}_Y(\mathrm{y})\, dF_Y(\mathrm{y}) = \int_0^\infty \int_y^\infty z-y dF_Y(\mathrm{z}) dF_Y(\mathrm{y}) \qquad (5)$$

$D_{YZ}$ and $D_Y$ here are the basic forms of housing relative deprivation index. Amendments of basic forms would be conducted before employing the index.

## 2.2. Improvement of the model

It is an obvious disadvantage of formula (1) ignoring the actual feeling of households, since it includes two cold quantities only. According to *Runciman* (1966) [4], "… the degree of a relative deprivation is the intensity with which it is felt." So it seems reasonable to improve the formula (1) as follows:

$$\mathrm{D}'(\mathrm{y},\mathrm{z}) = \begin{cases} U(z)-U(y) & y<z \\ 0 & y \geq z \end{cases} \qquad (6)$$

where $\mathrm{U}(\cdot)$ represents the utility of residential level to households, thus $\mathrm{D}'(\mathrm{y},\mathrm{z})$ represents housing relative deprivation felt by households. Apparently, $\mathrm{U}(\cdot)$ is an increasing function, and satisfies $U(x) \geq 0$, $\forall \mathrm{x} \geq 0$.

The formula (6) was proposed in *Hey* and *Lambert's* paper (1980) [5], which however did not make an in-depth analysis of $\mathrm{U}(\cdot)$. As an improvement, properties and formula of $\mathrm{U}(\cdot)$ will be discussed here. Two properties of $\mathrm{U}(\cdot)$ is concluded from the following two scenes which reveal feelings of households in terms of relative deprivation.

***Scene 1.*** Two cases are compared here with ***y=5, z=10*** and ***y=45, z=50.*** Same amount of ***y*** and ***z*** increase in this scene. There is obviously difference between the relative deprivations of household ***y*** in two cases. Degree of relative deprivation is lower than that in the former because

of the diminishing marginal utility. Formula (1) is unable to reveal difference of relative deprivation in these two cases.

***Scene 2.*** Two cases are similarly compared here with ***y=5, z=10*** and ***y=30, z=60.*** Same growth rate of ***y*** and ***z*** occurs here. It is reasonable that household ***y*** feels a higher degree of relative deprivation in the case that ***y=30, z=60*** than the former.

According to these scenes, U( ·) should satisfy the following properties,

(a) $\mathrm{U}(\mathrm{x}_1 + m) - \mathrm{U}(\mathrm{x}_2 + m) < \mathrm{U}(\mathrm{x}_1) - \mathrm{U}(\mathrm{x}_2)$， $\forall \mathrm{m} > 0$， $\mathrm{x}_1 > \mathrm{x}_2$.

(b) $\mathrm{U}(\mathrm{kx}_1) - \mathrm{U}(\mathrm{kx}_2) > \mathrm{U}(\mathrm{x}_1) - \mathrm{U}(\mathrm{x}_2)$， $\forall \mathrm{k} > 1$， $\mathrm{x}_1 > \mathrm{x}_2$.

The formula U( ·), which satisfies the properties above, is not the only one. However, we need to determine one formula in order to calculate housing relative deprivation quantitatively. Actually, there is a hypothesis in *Hey* and *Lambert's* model (1980) [5] that the formula U( ·) is $\mathrm{U}(\mathrm{x}) = \mathrm{x}$, which however does not satisfy the property (a). The formula as $\mathrm{U}(\mathrm{x}) = k \cdot \ln(\mathrm{x} + m)$ is employed in this paper. Specifically, $\mathrm{k} = 1, \mathrm{m} = 1$ is set here, that is,

$$\mathrm{U}(\mathrm{x}) = \ln(\mathrm{x} + 1) \qquad (7)$$

Which satisfies the properties above.

Then, since $\mathrm{D}(\mathrm{y}, \mathrm{z})$ is reformulated as $\mathrm{D}'(\mathrm{y}, \mathrm{z})$, the indexes $\mathrm{D}_{YZ}$ and $\mathrm{D}_Y$ are also need to be reformulated as,

$$\mathrm{D}'_{YZ} = E\big(\mathrm{D}_{YZ}(\mathrm{y})\big) = \int_0^\infty \mathrm{D}_{YZ}(\mathrm{y})\, dF_Y(\mathrm{y}) = \int_0^\infty \int_y^\infty U(z) - U(y) dF_Z(\mathrm{z}) dF_Y(\mathrm{y}) \qquad (8)$$

$$\mathrm{D}'_Y = E\big(\mathrm{D}_Y(\mathrm{y})\big) = \int_0^\infty \mathrm{D}_Y(\mathrm{y})\, dF_Y(\mathrm{y}) = \int_0^\infty \int_y^\infty U(z) - U(y) dF_Y(\mathrm{z}) dF_Y(\mathrm{y}) \qquad (9)$$

So far, housing relative deprivation index (HRD) has been constructed. When the distribution function of residential level is given, HRD $\mathrm{D}'_{YZ}$ and $\mathrm{D}'_Y$ could hence be calculated. Index $\mathrm{D}'_{YZ}$ reflects the housing relative deprivation of the households in group ***Y*** comparing with group ***Z***. When the reference group changes, HRD of the target group will change accordingly. Specifically, index $\mathrm{D}'_Y$, which is mainly employed in the following sections, reflects the housing relative deprivation of the households in group ***Y*** comparing with itself, indicating the average degree of housing fairness within a group. The higher the housing relative deprivation index, the lower the degree of housing fairness felt by households in group ***Y***.

# 3. HRD based on the current residential level

According to the model above, the distribution function of residential level in a group is the foundation to calculate HRD. Two distributions employed in this paper are constructed separately with two factors, one is the current residential level (represented by the current per capita floor area), and the other is the reachable residential level under the impact of household income.

HRD of China's urban households analyzed in this section is calculated based on the distribution of current residential level. This is a comparatively simple example of HRD calculation. And the complex calculation of HRD based on the reachable residential level will be showed in the next section.

The data in this section are from the fifth (year 2000) and the sixth (year 2010) national censuses. This data of urban household follow distribution of per capita floor area in provinces. Continuous distribution of data of provinces are discretized as Table 1 to simplify the calculation.

**Table 1. Per capita floor area distribution of national urban households**

| year | 4 | 10.5 | 14.5 | 18 | 24.5 | 34.5 | 44.5 | 55 |
|---|---|---|---|---|---|---|---|---|

| 2000 | 11.6% | 12.8% | 14.6% | 8.3% | 25.8% | 12.2% | 6.0% | 8.7% | | |
|---|---|---|---|---|---|---|---|---|---|---|
| year | 4 | 10.5 | 14.5 | 18 | 24.5 | 34.5 | 44.5 | 54.5 | 64.5 | 75 |
| 2010 | 8.4% | 6.4% | 8.6% | 5.8% | 23.4% | 17.0% | 11.0% | 5.8% | 4.6% | 9.0% |

Firstly, the data is considered as a group nationally to calculate the national average HRD. The HRD of urban households in China is 0.375 in 2000 compared with the national average distribution, while the figure increases to 0.397 in 2010. The 6 percent rise of HRD indicates that the degree of housing fairness decreased a little in the past ten years along with the improvement of residential level.

Next, each province is considered as a group with reference to itself to calculate the provincial average indexes. HRD of provinces are remarkably different (Figure 1). The HRD of Fujian is 0.483 in 2010, which is the highest of all, while the lowest one is 0.308 of Shandong that year. Interestingly, Fujian and Shandong are both at Eastern China, which indicates there is a significant difference of housing fairness among eastern provinces.

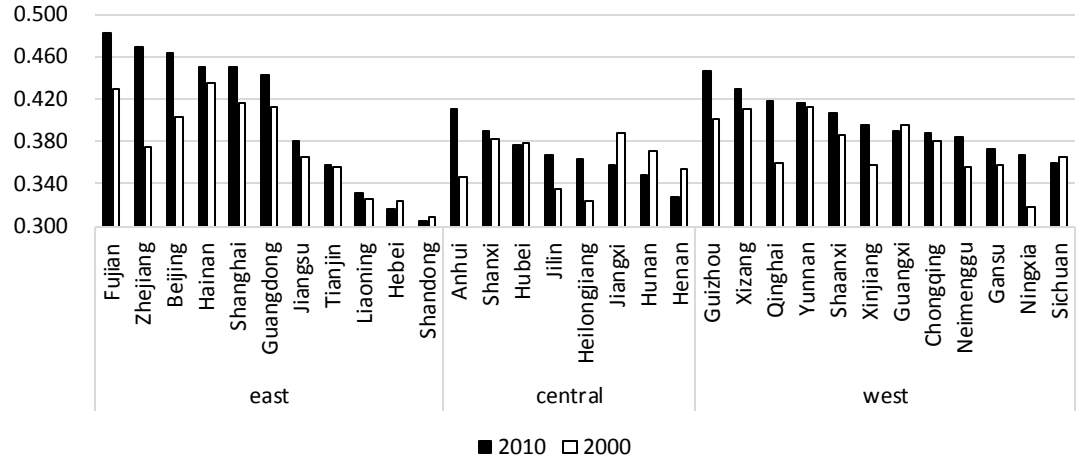


**Figure 1. Housing relative deprivation of provinces with reference to itself**

In addition, there are 23 out of 31 provinces suffering the increase of housing relative deprivation, among which there are 9 provinces with an increase by more than 10 percent. They are Zhejiang (24.8%), Anhui (18.2%), Qinghai (16.5%), Ningxia (15.3%), Beijing (14.9%), Fujian (12.5%), Heilongjiang (12.5%), Guizhou (11.3%), and Xinjiang (10.7%). Almost half of the provinces, Qinghai, Ningxia, Guizhou and Xinjiang, come from Western China.

The figures above indicate that housing fairness became noticeably worse in Western China in the past ten years. The state may result from the rapid economic development and serious uneven distribution of housing in this area. As a result, only a part of people but not all get rich in the progress of the West Development.

Besides, there are 3 provinces with a decrease by more than 5 percent: Jiangxi (-7.7%), Henan (-7.4%) and Hunan (-5.9%), which are all the central provinces. According to Figure 1 and these results, Central China is the area with the highest degree of housing fairness.

To conclude, the housing relative deprivation of urban households in China is calculated and analyzed above, reflecting the degree of fairness of current (year 2010) housing allocation. However, this is just an outcome analysis that ignored the potential change of residential level. When considering the potential change, the badly housed household may now be able to improve the residential level with high household income. The housing relative deprivation and the degree

of housing fairness cannot be described comprehensively through this outcome analysis. In order to evaluation housing fairness all sidedly, the future potential change of residential level should be considered.

# 4. HRD based on the reachable residential level

## 4.1. Model building

HRD based on ***the reachable residential level*** is constructed in this section, considering the impact of household income, housing price, etc. The reachable residential level is the residential level achieved by using part of the household income to improve housing conditions based on the current residential level.

Specifically, the model is constructed with the foundation of two-dimensional joint distribution function $F_{XY}(\mathrm{x},\mathrm{y})$ of current residential level $\boldsymbol{X}$ and household income $\boldsymbol{Y}$. And the reachable residential level $\boldsymbol{W}$ of a household is given by,

$$\mathrm{W} = \mathrm{X} + f(\mathrm{Y}, P) \qquad (10)$$

where $\boldsymbol{P}$ means housing price, and $f(\mathrm{y},\mathrm{P})$ means the improved residential level when the income of a household is $\boldsymbol{y}$ and the housing price is $\boldsymbol{P}$.

However, there is apparently no adequate opportunities for all households to improve residential level in housing market. So only household with income more than $\boldsymbol{y}^*$, which is the income threshold, has the ability to improve residential level, that is,

$$f(\mathrm{y},P) = \begin{cases} h(y,P) & y \geq \mathrm{y}^* \\ 0 & y < \mathrm{y}^* \end{cases} \qquad (11)$$

where the income threshold $\boldsymbol{y}^*$ is determined by housing price, mortgage interest rate, household preference, etc.

So far, the distribution function $F_W(\cdot)$ of reachable residential level $\boldsymbol{W}$ can be derived from $F_{XY}(\mathrm{x},\mathrm{y})$ and formula (10), when the housing price $\boldsymbol{P}$ and the income threshold $\boldsymbol{y}^*$ is given. Then the housing relative deprivation $\boldsymbol{D_W}$ based on reachable residential level can be calculated by using the model constructed in section 2.

## 4.2. Calculation of HRD $D_W$ - Beijing as an example

As an example, we will calculate the housing relative deprivation of Beijing based on the reachable residential level in this section. The data of city household per capita floor area are from the sixth national census. And the data of city household income are from sample survey of the basic situation of urban residents in 2010. It is important to note that the scope of studying object here is ***city***, different with ***urban*** in section 3. Two results from section 3 and 4 can therefore hardly be compared due to the sample difference.

The calculation progress includes mainly four steps: 1) to estimate the two-dimensional joint distribution function $F_{XY}(\mathrm{x},\mathrm{y})$; 2) to determine the income threshold $\boldsymbol{y}^*$; 3) to estimate the distribution function $F_W(\cdot)$; 4) to calculate the HRD $\boldsymbol{D_W}$.

**Step 1. To estimate the two-dimensional joint distribution function $F_{XY}(\mathrm{x},\mathrm{y})$**

The estimation of $F_{XY}(\mathrm{x},\mathrm{y})$ is not discussed in depth in this paper. Instead, we refer to the research achievement of Zheng Siqi, etc. (2013) [9], in which a two-dimensional joint distribution function $F_{XY}(\mathrm{x},\mathrm{y})$ is constructed. Figure 2 shows a two-dimensional joint probability density function $f_{XY}(\mathrm{x},\mathrm{y})$, which is differential of $F_{XY}(\mathrm{x},\mathrm{y})$.

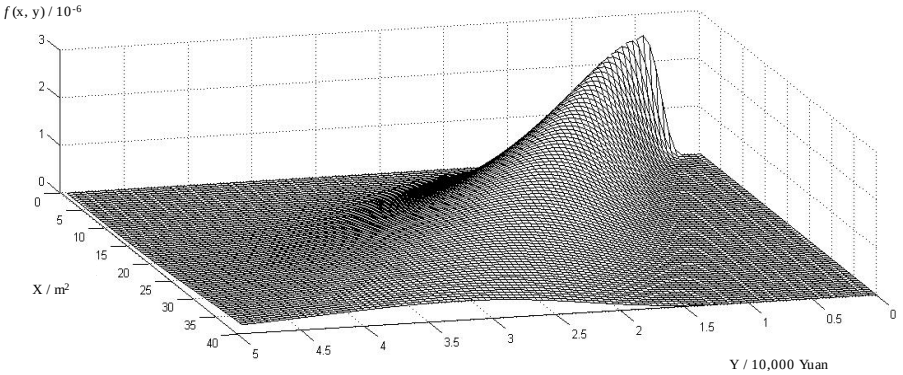


**Figure 2. Two-dimensional joint probability density function $f_{XY}(\mathbf{x},\mathbf{y})$**

**Step 2. To determine the income threshold $y^*$**

As said above, the income threshold $y^*$ is determined by housing price, mortgage interest rate, household preference, etc. Since the income threshold indicates the lowest income level to have access to commercial housing, a Minimum Residence, the total price of which indicates the lowest total price of commercial housing, is used to calculate the income threshold. And the total price of Minimum Residence is the product of the average housing price and the minimum housing area.

The minimum housing area is estimated from the maximum housing security level and the average household size. Specifically, per capita housing area of the maximum housing security level in Beijing is 15 square meters per person. And the average household size in Beijing city is 2.4 persons in 2010. So the minimum housing area is 36 ($= 15 \times 2.4$) square meters. In addition, the average housing price in Beijing is 17,151 Yuan per square meter[1]. Hence, the total price of Minimum Residence is about 617 ($= 17{,}151 \times 36 \div 1000$) thousand Yuan.

Then there are some general assumptions about mortgage and household preference for the calculation of income threshold. The rate of down payment is 30%. The interest rate is 6.14%. And the mortgage loan will be paid in equal installments over 360 months. In addition, the proportion of monthly payments in household income is assumed to be 50%.

According to the data above, the income threshold is 62,016 Yuan per year. In other words, if the annual income of a household is less than 62,016 Yuan in 2010, the household will not have the capacity to buy commercial housing, and will have no opportunities to improve residential level.

**Step 3. To estimate the distribution function $F_W(\cdot)$**

After the income threshold is determined, we are going to estimate the distribution function of the reachable residential level $\boldsymbol{W}$. According to the model above, the reachable residential level includes two parts: the current residential level $\boldsymbol{X}$ and the improvement $f(\mathrm{Y},P)$. Based on the determined income threshold and the average housing price, we can obtain the housing improvement function $f(\mathrm{Y},P)$ or $f(\mathrm{Y})$ (Figure 3), when the housing price $\boldsymbol{P}$ (17,151 Yuan) is given. As stated above, when the annual household income is less than 62,016 Yuan, the dwelling improvement is 0. Conversely, the dwelling improvement appears a linear growth with income.

[1] Calculated according to the housing sales and the sales area of Beijing in 2010 published by China's National Bureau of Statistics.

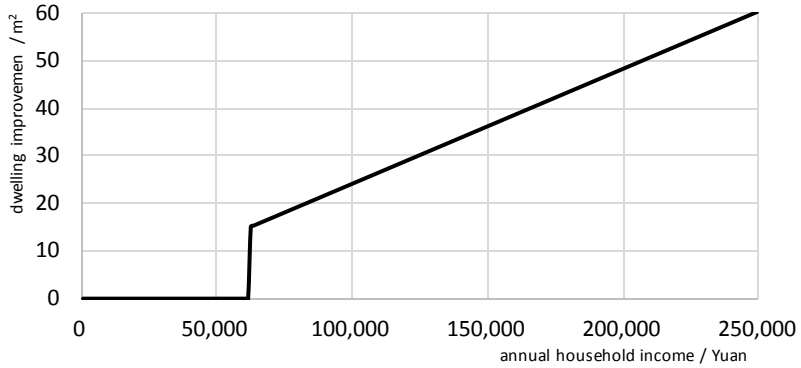


**Figure 3. The dwelling improvement function**

Then, the distribution function $F_W(\cdot)$ is got based on the two-dimensional joint distribution function $F_{XY}(\mathrm{x},\mathrm{y})$ and the dwelling improvement function $f(\mathrm{Y},P)$ (Figure 4). It is important to note that discretization is not employed here.

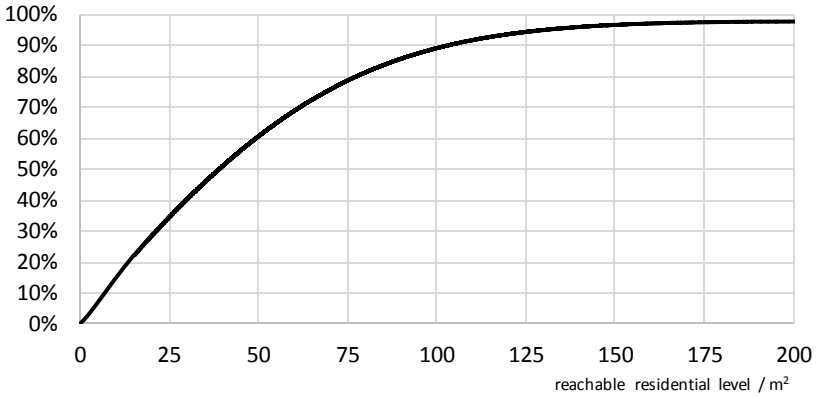


**Figure 4. The distribution function $F_W(\cdot)$**

**Step 4. To calculate the HRD $D_W$**

According to the model constructed above, the housing relative deprivation based on the reachable residential level can be calculated. And the HRD $D_W$ of city households in Beijing is 0.557 in 2010. In contrast, the HRD $D_X$, without consideration of income impact, is 0.510, less than the former. The result indicates that the matter of housing fairness becomes worse when the potential dwelling change is considered. It is mainly because there are positive correlation between the current residential level and household income. Generally, the better the current residential level, the higher the household income. So the gap of residential level between poor and rich gets wider when the household income is considered. In other words, this is a reflection of social unfairness in the field of housing.

# 5. Applications of HRD based on the reachable residential level

There are some significant applications of the HRD based on the reachable residential level, which is not just a cold and hard figure. One of the most important applications is to analyze the impact of various policies on housing fairness quantitatively, such as housing price control policy and housing security policy.

## 5.1. Impact of housing price control policy

It is always a criticized problem of high housing price, which is also one of the reasons for

the citizens to feel unfair in the field of housing. Thus, it is usually a measure of controlling housing price for China's government to show its effort on promoting housing fairness. Then, how does housing price fluctuation impact on housing fairness?

It is apparently the ability of housing improvement not the current residential level that would be influenced by the change of housing price. The decrease of housing price will impact household's ability on housing improvement from two aspects intuitively (Figure 5).

1) The income threshold will decrease resulting in the increased opportunities for low-income households to purchase commercial housing. Then the housing relative deprivation of these households will decrease. And the degree of housing fairness will increase in consequence.

2) The housing improvement ability of high-income households will increase so that the gap of residential level between the low-income and the high-income will be wider. Then the housing relative deprivation of the low-income households which still have no abilities to buy commercial housing will increase. And the degree of housing fairness will decrease in consequence.

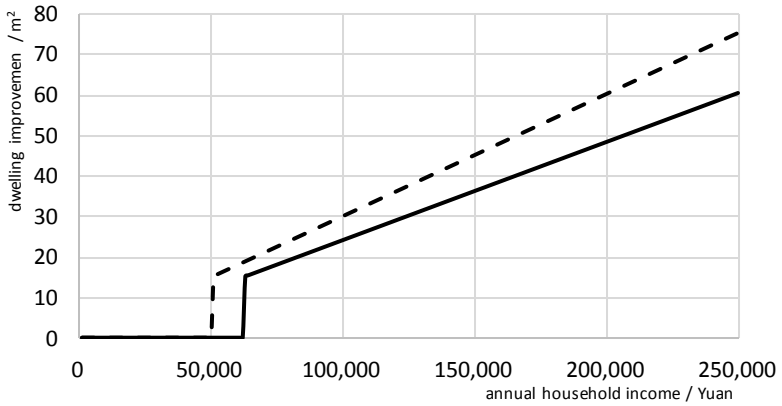


**Figure 5. The impact of housing price decrease on housing improvement ability**

Impacts on housing fairness degree from these two aspects are converse. So quantitative calculation is necessary to estimate the impact of housing price movement more accurately. Based on the model constructed above, we calculate the HRD at various levels of housing price (Figure 6). The **x** axis is various percent of the current (year 2010) average housing price. For example, 100% represents that the housing price is equal to the current average housing price (=17,151 Yuan per square meter). And 80% indicates 20% decrease of housing price (=13,721 Yuan per square meter). Dashed line here is the level of housing relative deprivation (=0.510) without consideration of income impact.

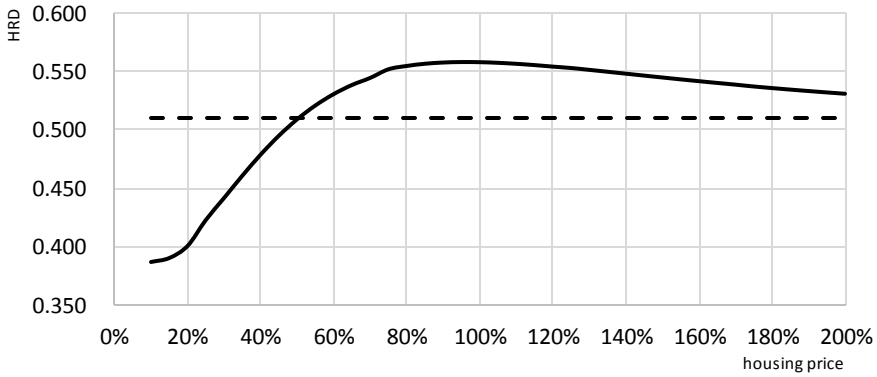


**Figure 6. HRD at various levels of housing price**

The curve of housing relative deprivation presents an inverted U-shaped trajectory along the movement of housing price. And the curve can be divided into 4 parts. 1) When housing price is low, the reachable residential levels of most households are all high. And the HRD is low, thus the degree of housing fairness is high. 2) On the left hand side of the critical point ($\approx 90\%$), the HRD becomes higher along with the increase of housing price. It is due to the highlight of the difference between the changing abilities of the high-income and the low-income households in improving residential level. 3) On the right hand side of the critical point, the HRD decreases along with the increasing housing price. At this stage, the gap between reachable residential levels of the low-income and the high-income households becomes narrower, because the housing price is so high that the improvement ability of the high-income households is also affected. 4) The limiting case is that housing is too expensive to buy for all households regardless of whether the household is low-income or high-income. Then the HRD here is equal to the one (=0.510) without the consideration of household income impact.

It is seen from Figure 6 that the current (year 2010) status is at the third part, which indicates that it cannot improve housing fairness effectively by decreasing housing price slightly (10%~20%). The discount of housing price has to be at least 30% to pull down the housing relative deprivation effectively. What's more, if the relative deprivation is expected to be less than 0.510, the discount has to be more than 50%, which is a substantial decrease. In other words, if it is purposed to improve housing fairness relying solely on housing market, the housing price will keep on a low level.

## 5.2. Impact of housing security policy

The impact of housing price movement on housing fairness is calculated quantitatively above by using the housing relative deprivation index. And it is found that the degree of housing fairness improves little by the slight decrease of housing price.

Actually, housing price control is just one of the methods to improve housing fairness, and the alternative method is housing security, which will be discussed in this section. As we all know, there are many different housing security policies in China. Then, under same government input level, which housing security policy may improve condition of housing fairness more efficiently? The impacts of two different policies, the owner-occupied housing policy and the low-cost social housing policy, will be compared by HRD in the next moment.

The owner-occupied housing policy is purposed to provide housing for the low-middle-income households at favorable prices in order to enhance their housing improvement ability. This policy is simplified in the model as,

*If the household, with annual income less than 62,016 Yuan, has no ability to buy commercial housing, the household is then qualified to access owner-occupied housing at the average market price of a **K** discount.*

The low-cost social housing policy is purposed to provide low-rent housing for the badly housed low-income households. The policy is simplified in the model as,

*If the annual income of the household is less than **M** Yuan, and the per capita housing area is less than **S** square meters, the household is qualified to obtain a low-rent housing to reach the per capita housing area of **N** square meters.*

It can be seen that the total cost of the one policy, which is expended by the government, may be different from the other when the parameters ***K, M, S, N*** take different values. Then, it makes no sense to compare the housing fairness improvement (output) of the two policies in the case of

different cost (input). So the equal input (formula 12) is the foundation to compare the output of the two policies.

*The input of owner-occupied housing ($C_1$) = the input of low-cost social housing ($C_2$)* (12)

Further speaking, the input of a housing security policy could be represented by the product of the number of security housing, which is the number of households benefited from the policy ($H_1$,$H_2$), and the average cost of per unit security housing ($B_1, B_2$). So the formula (12) can be transformed through the following derivation.

$$H_1 \times B_1 = H_2 \times B_2 \qquad (13)$$

$$H_1 = H_2 \times \frac{B_2}{B_1} \qquad (14)$$

The value of $\frac{B_2}{B_1}$ can be estimated as follows. The cost of per unit security housing is a product by the subsidized price difference and the unit area. In terms of the subsidized price difference, the subsidy by the government for per unit low-cost social housing is almost equal to the average housing price $\boldsymbol{P}$ due to the very low rent. And the subsidy for per unit owner-occupied housing is about $\boldsymbol{K \times P}$**.** In terms of the unit area, the per capita housing area of low- cost social housing is assumed to be 10 square meters, that is $\boldsymbol{N = 10}$, in order to simplify the calculation. And the per capita housing area of owner-occupied housing is about 20 square meters[2]. In addition, the household size is 2.4. Then the expression of $\frac{B_2}{B_1}$ can be derived as,

$$\frac{B_2}{B_1} = \frac{P \times N \times 2.4}{Z \times P \times 20 \times 2.4} = \frac{1}{2 \times Z} \qquad (15)$$

So far, in order to equalize government input on the two policies, $H_1$,$H_2$ should satisfy the following formula,

$$H_1 = \frac{1}{2} \times \frac{H_2}{Z} \qquad (16)$$

The appropriate parameters can be determined based on the two-dimension joint distribution $F_{XY}(x,y)$ in order to satisfy the formula (16). Some comparable cases are listed in Table 2. The input of the first case is standardized as 1.0, and the input of the other cases can be calculated accordingly.

**Table 2. Some comparable cases**

| government input | | the owner-occupied housing | | the low-cost social housing | | |
|---|---|---|---|---|---|---|
| | | ***K*** | proportion of $H_1$ | ***M*** | ***S*** | proportion of $H_2$ |
| low | 1.0 | 10% | 5.6% | 12500 | 6.25 | 1.1% |
| ↓ | 2.3 | 15% | 8.7% | 17500 | 7.25 | 2.6% |
| ↓ | 4.2 | 20% | 11.8% | 23500 | 7.25 | 4.7% |
| ↓ | 6.8 | 25% | 15.2% | 30500 | 8.75 | 7.6% |
| high | 10.6 | 30% | 19.8% | 43500 | 9.75 | 11.9% |

Base on the comparable cases selected, the housing relative deprivation index separately under the two policies could be calculated. The result is shown in Figure 7, the x axis in which is the standardized government input. In addition, line ***HRD-A*** shows the impact of owner-occupied

[2]The per capita housing area of owner-occupied housing here is determined to guarantee the available housing for all protected households. Thus, the per capita housing area is actually related to the value Z. But the paper here assumed the per capita housing area to be 20 square meters, which is the average value when Z changes from 10% to 50%.

housing policy on the housing relative deprivation, while line ***HRD-B*** stands for the low-cost social housing policy.

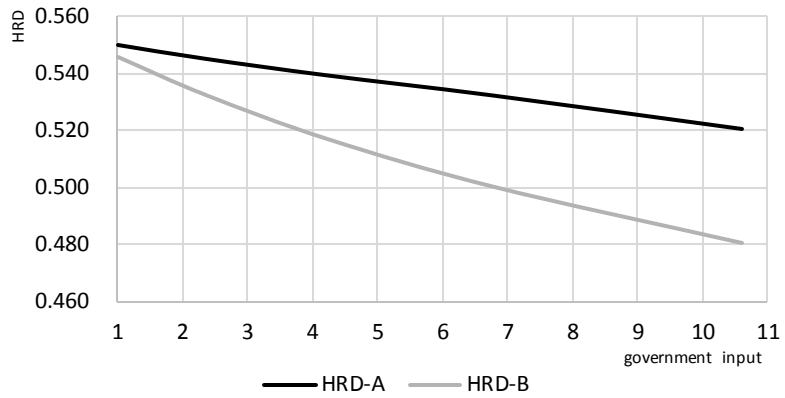


**Figure 7. The impact of the two policies on HRD**

It can be seen obviously that HRD under both policies are lowered, which means improvement of housing fairness comes along with larger government input. And the impact of the owner-occupied housing policy is less than that of the low-cost social housing policy under same input of government. According to Figure 7, the cost of the owner-occupied housing policy will be 2~3 times than the other for reaching the same degree of impact. In conclusion, it is for the owner-occupied housing policy that the proportion of benefited households is larger, and the efficiency to improve housing fairness is lower than those of the low-cost social housing policy.

# 6. Conclusion

This paper introduces relative deprivation as a quantitative measurement of fairness to the field of housing, and constructs housing relative deprivation index (HRD) based on the current residential level as well as the reachable residential level. Then the impacts of different housing policies, such as housing price control policy and housing security policy, on housing fairness are analyzed quantitatively by adopting HRD which based on the reachable residential level.

According to HRD based on the current residential level, Central China is the area with the highest degree of housing fairness, and the significant worse of housing fairness occurred in Western China in the past ten years (2000-2010).

Based on the reachable residential level, HRD is adopted to evaluate impacts of housing policies. For housing price control policy, HRD indicates that it can only enhance housing fairness by decreasing housing price for at least 30%, but not slightly decreasing for 10%~20%. For housing security policy, the result indicates that the efficiency of low-cost social housing policy to improve housing fairness is higher than that of owner-occupied housing policy under the same government input. And if it is purposed to reach the same degree of impact, the government input on the owner-occupied housing policy will be 2~3 times than the other.

Since it is a primary attempt for this paper to evaluate housing fairness quantitatively, some improvements need to be made in future research. Above all else, the deeper research of fairness perception is vital to the quantitative evaluation of fairness. In other words, U(.) is the foundation of quantitative evaluation. The formula $U(x) = \ln(x + 1)$ employed in this paper is a strong assumption according to properties. If the formula U(.) is changed, all results would be changed accordingly. While the methods constructed in this paper are still applicable.

# Reference

[1]Rawl, J. (1971). *A theory of justice*. Harvard University Press.

[2]Liu Hongyu, Yang Fan, & Xu Yuejin. (2013). Housing Distribution in Urban China Basing on China's 2010 Census. *Journal of Tsinghua University (Philosophy and Social Sciences)*, (6), 138-147.

[3]Marx, K., & Engels, F. (1902). *Wage-labor and Capital*. New York Labor News Company.

[4]Runciman, W. G. (1966). *Relative Deprivation and Social Justice: A Study of Attitudes to Social Inequality in Twentiety-century England*. University of California.

[5]Hey, J. D., & Lambert, P. J. (1980). Relative deprivation and the Gini coefficient: comment. *The Quarterly Journal of Economics*, 567-573.

[6]Yitzhaki, S. (1979). Relative deprivation and the Gini coefficient. *The Quarterly Journal of Economics*, 321-324.

[7]Betti, G., & Verma, V. (2008). Fuzzy measures of the incidence of relative poverty and deprivation: a multi-dimensional perspective. *Statistical Methods and Applications*, *17*(2), 225-250.

[8]Bárcena-Martín, E., Imedio-Olmedo, L., & Martín-Reyes, G. (2007). Inequality and deprivation within and between groups: An illustration of European union countries. *The Journal of Economic Inequality*, *5*(3), 323-337.

[9]Zheng Siqi, Sun Weizeng, & Xu Yangfei. (2013). Estimation methodology of public housing coverage ratio and its application in Chinese cities. Working paper.