Abstract

Empirical evidence suggests that when the market becomes increasingly volatile, trading activities may be depressed or even halted. This paper develops a model and formally studies the relationship between the market volatility and its liquidity in the real estate market. Different information structures are examined in the context of a seller-offer, ultimatum bargaining game. We show that an increase in the market volatility negatively affects its liquidity when information is asymmetric between the buyer and the seller. However, market volatility has no effect on liquidity under symmetric information when the buyer and the seller are either both informed or both uninformed. We extend our common-value model to the case of private value, whereby the main findings of Haurin’s (1988) classic work are reaffirmed under an alternative model setting.

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1. Introduction

The real estate market distinguishes itself from the financial market in its high degree of illiquidity. Unlike traders in the financial market who can readily buy or sell a security at its equilibrium price, both the buyer and seller in the real estate market must spend time and exert search effort in order to find a suitable counter party and complete the transaction, largely due to the heterogeneity of different properties. In addition, the outcome of an attempted transaction is often uncertain despite the traders’ good faith and/or the seller’s best marketing effort. In some circumstances the resulting probability of trade may be less than one leaving the property unsold eventually even though *ex post* gains from trade exist. The question of what determines real estate liquidity has been researched extensively in the literature. Genesove and Han (2012) enumerate the extant studies on the determinants of real estate liquidity, among them: idiosyncrasy of the property (Haurin, 1988), seller motivations (Glower et al, 1998), initial offer price (Anglin et al, 2003), owner equity (Genesove and Mayer, 1997), previous purchase price (Genesove and Mayer, 2001), initial list price (Anglin et al, 2003), and brokers (Levitt and Syverson, 2005; Hendel et al, 2009; and Bernheim and Meer, 2008). This line of work highlights the multifaceted nature of the liquidity issue, and has prompted researchers to look further into the problem for other factors that may affect real estate liquidity.

In this paper we investigate the role of market volatility in the determination of real estate liquidity. The real estate market, being prone to the influence of various

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1 These papers focus on seller time on the market (STOM). Papers that focus on buyer time on the market (BTOM) are also listed in Genesove and Han (2012), with the latter itself offering an equilibrium analysis of both STOM and BTOM.
policy shocks and other economic forces, is inherently volatile. There is some empirical evidence which suggests that when the market is volatile and agents become increasingly uncertain about the future trend of certain economic variables relevant to their decision making, market activities might be depressed or even halted. In their report on the 1997 East Asian financial crisis, J.P. Morgan (1998) states that “the lack of property transactions [in the real estate market] reflects investor uncertainty” (Jones and Manuelli, 2002). In a similar vein, Jones and Manuelli (2001) quote Heymann and Leijonhufvud (1995) in their description of the hyperinflation situation and the related market-wide price uncertainty that occurred in Argentina during the late 1980s:

*a customer finds a good inside a shop, with a clearly marked price, and decides to buy it. The shopkeeper refuses; he explains that the posted price has no significance, because he cannot be sure that the wholesaler will not double his own price the next day. When asked what he would do if someone offered to pay double the marked price, the shopkeeper answers that he would not sell anyway, for what if the wholesale price tripled before he replaced the good?*

Implicit in the above account is the recognition that the customer might have superior information about the next day’s price movement which the shopkeeper does not have. As a result the shopkeeper becomes wary of the customer’s motive for trade and refuses to trade with her - a typical adverse selection problem. In situations like this, volatile market conditions coupled with agents’ information asymmetry regarding

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2 For studies on the determinants of house value volatility, see Zhou and Haurin (2010) and the references cited therein.
certain economic variables of the environment may thus produce a negative impact on the market activity.

The real estate market, by its very nature, is one plagued by asymmetric information. The notion of information asymmetry finds its place in much of the research on real estate transactions. For example, in De Wit and Van der Klaauw (2010), the seller has private information about certain attributes of his house and/or his own characteristics (e.g., risk preference, financial constraints, and degree of patience). Taylor (1999) models a situation of two-sided asymmetric information in which the seller privately observes the quality of his house while the buyer’s taste for the house is known only to the buyer herself. In two influential papers, Rutherford, Springer and Yavas (2005, 2007) find that real estate agents on average receive a higher sale price on their own houses than on similar houses owned by their clients, which the authors interpret as evidence for the existence of asymmetric information between the house seller and his agent. These different forms of asymmetric information in the real estate market, whether seller-sided, buyer-sided, or in a seller-agent relationship, conforms well to one’s intuition about the market: The seller inherently knows more about his own house and/or about himself than the buyer does, the buyer’s tastes for and private valuation of the house are typically unobservable to the seller, and the real estate agent by virtue of her specialized knowledge and expertise possesses better information than her client when it comes to setting ask price and negotiating with the buyer.

The current paper examines the effect of market volatility on the liquidity of real
estate properties in the presence of asymmetric information. We start with the benchmark case of symmetric information where the buyer and the seller are either both informed or both uninformed about the fundamental value of the property for sale. We find that market volatility is “neutral” with respect to its effect on property liquidity: the probability of sale is always one regardless of the degree of market volatility. We then consider the case of an informed buyer trading with an uninformed seller. In this situation common value learning takes place on the part of the seller over the entire marketing period, and the seller’s optimal search strategy is characterized by a monotonically decreasing sequence of offer prices to a set of potential buyers. We explicitly derive the optimal price sequence whereby an inverse relationship is established between the market volatility and the liquidity of the property. To get a complete picture of the volatility-liquidity relationship the complementary case of an uninformed buyer trading with an informed seller is also examined. We characterize the perfect Bayesian equilibrium (PBE) of the trading game where the seller signals his private information about the property’s fundamental value to the buyer through his offer price. A plausible equilibrium selection criterion allows us to narrow down the set of PBEs of the signaling game. In the equilibrium that maximizes the seller’s expected payoff, which at the same time also maximizes the probability of trade, the liquidity of the property is shown to be negatively correlated with the market volatility.

Our paper is most closely related to the work by Krainer (2001), in which the author presents a theory of liquidity in the residential real estate market. One of the
main purposes of the theory is to explain a stylized fact commonly observed in the
market, namely, that real estate liquidity varies substantially over time and across
different states of nature. Using a search-theoretic model where liquidity and price
emerge endogenously as a consequence of the traders’ optimizing behavior, Krainer
(2001) shows that market liquidity is higher in the “high state” than it is in the “low
state”. The high and low states refer to the relative magnitude of an aggregate value
component in the property’s dividend process, akin to the fundamental value
component in the decomposition employed in our model. Krainer (2001) uses a
Markov chain process to model the evolution of the aggregate value component which
can take on two values, high and low, with the high (low) value representing the high
(low) state of the real estate market. In essence Krainer (2001) identifies a level effect
of the market-wide value component but he does not consider the effect of the
volatility of this component on the market liquidity. Our results complement those of
Krainer (2001) by focusing on the other aspect of the issue, namely, how the market
volatility affects the real estate liquidity. While Krainer (2001) obtains a positive
state-liquidity relationship, we derive a negative volatility-liquidity relationship in the
cases of asymmetric information. One important feature shared by Krainer (2001) and
our paper is the measure of liquidity used in the analysis. Instead of the
time-on-market (TOM), a commonly used measure of liquidity in most other studies,
both of our papers invoke the probability of trade as an alternative measure of
liquidity as the latter is better suited to the particular analytic framework adopted in
the two papers. As Krainer (2001) argues, “[t]he equilibrium liquidity is synonymous
with the equilibrium probability of sale. Thus, the most obvious measure of liquidity is the probability-of-sale function...”

Our paper is also related to the classic work of Haurin (1988). In contrast to the common-value setting considered in our paper, Haurin (1988) studies the marketing problem of a seller in a private-value setting where the potential buyers’ private valuation of the “atypical” attributes of the property is unknown to the seller. Adopting an optimal-stopping rule paradigm Haurin (1988) establishes an inverse volatility-liquidity relationship in his model: atypical houses, whose value volatility is greater than that of more standard houses, tend to stay on the market longer. In the last part of our paper we extend our analysis to study a variant of Haurin’s (1988) model in an attempt to provide some “robustness check” on the main findings of this classic work. Several modifications are made to the original model including the relative role played by the buyer and the seller in the bargaining game and an alternative way to model risk and volatility. Our findings from the modified model corroborate those of Haurin (1988), namely, in a setting characterized by private value and information asymmetry, the liquidity of the property is negatively correlated with the volatility of the property value.

The rest of the paper is organized as follows. Section 2 describes the basic model and derives the results for the benchmark cases of symmetric information. The cases of asymmetric information are considered in Sections 3 and 4 where the main findings of the paper are derived. Section 5 revisits Haurin’s (1988) problem, and Section 6 concludes.
2. The Real Estate Trading Game

We consider a real estate market in which the buyers and sellers may possess symmetric or asymmetric information about the market-level, fundamental value of the real estate property for sale. The real estate market goes through hot and cold periods during which property prices can fluctuate substantially. The fluctuations, in general, are not caused by changes in the buyers’ taste for the property, as taste is idiosyncratic and unlikely to vary much over time. Rather, property prices fluctuate mainly because market fundamentals change, as the asset pricing approach to real estate valuation suggests (Krainer, 2001). Moreover, prior to trading a real estate property, the information available to the buyer and the seller may well be different concerning the market fundamentals that will prevail in the future. To the extent that real estate is both a consumption good and an investment good (Henderson and Ioannides, 1987), and given that the real estate consumers’ preferences and tastes do not experience much change over time, it is the change in the fundamental value of the property that is more likely responsible for the constant price fluctuation in the market. In view of these observations, following Krainer (2001), we decompose the total value of a property into two components: a fundamental value component and an idiosyncratic value component. The idiosyncratic value component is buyer or seller specific and thus is independent across market participants. The fundamental value component is essentially a common value shared by all market participants, as opposed to the idiosyncratic value component which can be viewed as a private value.
for individual buyers and sellers. Krainer (2001) offers a nice interpretation of the fundamental value component of the property, ranging from land value to the state of the economy. Our property value decomposition is also reminiscent of the well-known house price decomposition proposed by Case and Shiller (1989) where they view the (log) house price as the sum of the city-wide (log) house price level, a property-specific component and a mean-zero random trading noise.

Suppose that there are two types of risk neutral agents in the real estate market: buyers and sellers. The sellers each have one unit of housing for sale, and the buyers each demand one unit of housing. Besides the housing good, the agents in the economy consume another good which we call general good. We consider a two-period model. In the first period, each type of agent receives a fixed endowment of money which they can use to buy either the housing good or the general good. The money endowments are denoted by \( M^b \) and \( M^s \) for the buyer and the seller respectively. Agents derive utility from consuming both goods in the second period, and we assume utility is additive over the two goods. Without loss of generality the price of the general good is normalized to be one. For mathematical simplicity, suppose that there is no time discounting in the economy.

As discussed above, the value of the housing unit consists of two parts: a fundamental value component and an idiosyncratic value component. Specifically, the total value of a house to the buyer and the seller can be expressed, respectively, as

\[ v^b = f + d^b \quad \text{and} \quad v^s = f + d^s. \]

We assume \( d^b > d^s \) so that gains from trade exist; without loss of generality we further assume \( d^b = d > 0 \) and \( d^s = 0 \). Hence
\[ \nu^b = f + d \quad \text{and} \quad \nu^s = f . \] This assumption implies that the buyer and the seller share a common-value component \( (f) \) in the house with regard to its fundamental value but differ in their idiosyncratic valuation of the property \( (d^b \; \text{vs.} \; d^s) \).\(^3\) Thus, when a buyer and a seller meet in the first period and decide to trade a housing unit at price \( p \), the second-period payoffs of the buyer and the seller are given by

\begin{align*}
\pi^b(p) &= \nu^b + M^b - p = f + d + M^b - p, \quad (1) \\
\pi^s(p) &= M^s + p. \quad (2)
\end{align*}

If the two parties decide not to trade, then their second-period payoffs are instead

\begin{align*}
\pi^b &= M^b, \quad (3) \\
\pi^s &= \nu^s + M^s = f + M^s. \quad (4)
\end{align*}

Note that the sum of \( \pi^b \) and \( \pi^s \) in the case of trading exceeds that in the case of no trading by a positive amount \( d > 0 \). This has the implication that trading the housing unit is always the efficient outcome.

As in Krainer (2001) the trading process is modeled as an ultimatum bargaining game in which the seller, based on all information (public and private) available to him, makes a take-it-or-leave-it offer to the potential buyer. If the buyer decides to buy the property at the seller’s offer price, the two parties obtain their respective

\(^3\) As argued by Yavas and Yang (1995) agents may value a property differently and thus have motivation for trade because they may have “different liquidity needs, different preferences, or different plans for future,” among other reasons (page 350). Our specification explicitly introduces a common-value component in the property value in order to capture the notion that certain attributes of a house (e.g., those related to its investment value) are typically valued the same by all agents, while still allowing for differences in their valuation of other attributes (e.g., those related to the house’s consumption value). The existence of gains from trade (i.e. \( d^s > d^b \)) is also assumed in Yavas and Yang (1995), as is common in most trading models of the real estate market.
payoffs in the second period as specified above. Otherwise, the seller searches for another buyer and the process starts anew. Following Haurin (1988), we assume the buyers’ arrival rate is constant: exactly one potential buyer arrives to visit the seller’s property per unit of time. One period in our model comprises a fixed number, $N$, of time units, during which the seller must either sell the property or exit the market.\(^4\) In other words, the seller has a maximum marketing horizon of fixed duration. During the marketing period the seller incurs a cost of holding the property. Let $g(T)$ be the total holding cost over the marketing period $T$. We assume $g(0) = 0$ and $g'(T) > 0$. This implies that the total cost of holding the property is strictly increasing with the time elapsed.

Our real estate trading model features both symmetric and asymmetric information between the buyer and the seller concerning the fundamental value of the house. We consider, in turn, different information structures and their implications for the relationship between market volatility and real estate liquidity. Specifically, we study four different information pairs: (a) informed buyer and informed seller, (b) uninformed buyer and uninformed seller, (c) informed buyer and uninformed seller, and (d) uninformed buyer and informed seller. In each of these cases, the informed party knows the house’s fundamental value exactly, while the uninformed party only has knowledge about the distribution of this variable, which for simplicity is assumed to be the uniform distribution over a finite interval: $f \sim U[\mu - k, \mu + k]$, with $\mu > k$.

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\(^4\) This assumption is similar to the one made in Read (1988) which conforms more closely to the reality of the real estate market and obviates the need to deal with an infinite marketing horizon.
The information structure itself is common knowledge.\textsuperscript{5} Note that the dispersion of the uninformed party’s assessment of the future fundamental value of the house is a consequence of volatile market conditions, and reflects the inherent uncertainty the uninformed party faces when making trading decisions. Thus $k$ is a natural candidate as the measure of market volatility, which we adopt in the subsequent analysis. To ensure interior solutions and avoid the trivial cases where the probability of trade is always one, we also assume $k > d$.

In this section we start with an analysis of the benchmark cases of symmetric information. We investigate the role of information structures (a) and (b) in determining the relationship between market volatility and its liquidity. First, consider the case where both the buyer and the seller are informed about the fundamental value of the house. When facing a price $p$ offered by the seller, the buyer’s problem is to decide whether or not to buy the house at the offer price. Since she knows the fundamental value of the house and hence knows the level of payoff she can obtain if she buys the house at the offer price, and since she can always obtain a payoff of at least $\pi^b = M^b$ by not trading with the seller (and consuming only the general good), the buyer will accept the seller’s offer if and only if $p \leq f + d$. Knowing this, the seller who is also informed about the fundamental value, will offer a price $p = f + d$. The buyer will accept the seller’s price offer and the property is traded with probability one in the seller’s first marketing attempt. In this equilibrium, efficiency is

\textsuperscript{5} The buyer’s idiosyncratic value component, $d$, is assumed to be known to both the buyer and the seller. This assumption is made so as to disentangle the effect of the uncertainty about the fundamental value; the case of unknown idiosyncratic value is considered in Section 5. On the other hand, the current assumption can be justified if the housing unit is sufficiently “typical” that a hedonic model can be employed to estimate the market value of the property’s attributes, making the latter almost common knowledge among all agents (Haurin, 1988).
achieved with the seller extracting all the surplus (equal to $d$) by virtue of his first-mover advantage.

Next consider the case where neither the buyer nor the seller are informed about the property’s fundamental value $f$. By assumption both parties’ common prior on $f$ is the uniform distribution $U[\mu - k, \mu + k]$. In this situation the seller maximizes his expected payoff, taking into account the buyer’s acceptance strategy. Because the buyer is risk neutral she accepts the seller’s offer if and only if $E[\pi^b(p)] \geq M^b$ or, equivalently, $p \leq \mu + d$. The equilibrium thus has the seller offering a price $p = \mu + d$. At this price the property is traded with probability one in the seller’s first marketing attempt. As in the previous case the equilibrium achieves ex ante efficiency, with the seller again extracting the entire surplus.

In both of the above cases, market volatility (measured by the dispersion parameter $k$) does not affect the probability of trade between the buyer and the seller. A neutrality result for these benchmark cases is summarized in the following proposition.

**Proposition 1.** If the buyer and the seller are symmetrically informed about a property’s fundamental value, in the ultimatum bargaining game between the two traders, market volatility has no effect on the probability of trade and hence no effect on the property’s liquidity. In equilibrium, the buyer and the seller trade the property with probability one in the seller’s first selling attempt.
3. Sequential Search with Buyer-Sided Private Information

In this section we turn to the case of an informed buyer trading with an uninformed seller, and examine the relationship between market volatility and asset liquidity under such an information structure. The buyer has private information about the fundamental value of the house for sale but the seller’s information set is one of public information, i.e., his \textit{ex ante} assessment of the property’s fundamental value is the commonly known uniform distribution $U[\mu-k, \mu+k]$.

We characterize the seller’s search behavior in such an environment. When the seller makes a take-it-or-leave-it price offer to a potential buyer, there are two possible outcomes: either the offer is accepted or it is rejected. Both are the outcomes of the buyer’s optimizing behavior given her private information about the fundamental value. Although too low a price will certainly be accepted by the buyer and result in a quick sale, the seller typically does not want to sell the house too quickly at the cost of a lower sale price. On the other hand, if his first offer price is set too high and rejected by a potential buyer, the seller will have to wait another unit of time and incur the corresponding holding cost before he has the chance to offer his house to a new buyer. There exists a trade-off between a prolonged marketing period and the resulting high holding cost, and fetching a high price on the property. In such circumstances the seller’s optimal search strategy is one of experimenting with a sequence of prices offered sequentially to a set of potential buyers.

Lazear (1986) pioneers the study of retail pricing strategy in an asymmetric information environment. In his model, the seller is uncertain about the value of his
product to the potential buyers. If the product fails to sell at his first price offer, the seller rationally updates his beliefs about the product’s market value and adjusts the price downward in his offer to a second buyer, and so on. He shows that for any marketing period of fixed length there exists an optimal sequence of offer prices that is monotonically decreasing over time. Although Lazear’s model is developed in the context of the retail business, the framework has been employed subsequently by several authors to study the search process in the real estate market, e.g., Read (1988) and An et al (2013).

We follow this literature and derive explicitly the seller’s optimal price sequence whereby the implications for market liquidity are analyzed. Our model differs from those in Lazear (1986), Read (1988) and An et al (2013) in one important respect. In all of the aforementioned papers, the seller has a valuation of the product that is independent of the buyer’s - a private-value setting. In this setting when the seller fails to sell the product in the first period, learning occurs on the part of the seller regarding the true value of the product to the buyer. The new information learned from the updating procedure has no bearing on the value of the product to the seller himself; it only improves the seller’s estimate of the product value to the buyer. Thus, this is a case of private-value learning. In contrast our model is one of common-value learning: failing to sell the real estate property provides the seller with new information, not only about the buyer’s valuation of the property but also about the seller’s own valuation, with the property’s fundamental value component as the common link.
To gain intuition we begin with the simplest case of just one price offer. Suppose the seller’s planned marketing duration is one unit of time, during which he can only make a price offer \( p_1 \) to one potential buyer. Since the (informed) buyer will accept the seller’s offer if and only if \( p_1 \leq f + d \), the seller’s expected payoff, if he offers \( p_1 \), is given by

\[
\pi^{(i)}(p_i) = M^* + p_1 \cdot \Pr^{(i)}\{p_i \leq f + d\} + \frac{1}{2}[(\mu - k) + (p_i - d)] \cdot \Pr^{(i)}\{p_i > f + d\}
\]

where the second term on the right-hand side of the equation reflects the fact that, if the seller’s offer is rejected by the buyer, the seller’s posterior belief about the property’s fundamental value \( f \) is the truncated uniform distribution \( f \sim U[\mu - k, p_i - d] \), which has an expected value \( \frac{1}{2}[(\mu - k) + (p_i - d)] \). Since, ex ante, \( f \sim U[\mu - k, \mu + k] \) we have

\[
\Pr^{(i)}\{p_i \leq f + d\} = \frac{\mu - k - (p_i - d)}{2k}, \tag{5}
\]

and

\[
\Pr^{(i)}\{p_i > f + d\} = \frac{p_i - d - (\mu - k)}{2k}. \tag{6}
\]

Substituting (5) and (6) into the expression for \( \pi^{(i)}(p_i) \) and simplifying, we get

\[
\pi^{(i)}(p_i) = M^* + p_1^2 + 2(\mu + k)p_1 - \frac{(\mu - k)^2 - d^2}{4k}.
\]

The seller’s problem is to choose \( p_1 \) to maximize his expected payoff, which yields the following first-order condition:

\[
\frac{\partial \pi^{(i)}(p_i)}{\partial p_i} = \frac{-2p + 2(\mu + k)}{4k} = 0
\]

The solution to this equation gives the seller’s optimal choice of offer price:

\( p_1^* = \mu + k \). We can then compute the corresponding probability of trade \( \theta^{(i)} \) and the
seller’s expected payoff $\pi^{(1)}(p^*_1)$ as

$$\theta^{(1)} = \Pr\{p^*_1 \leq f + d\} = \frac{\mu + k - (p^*_1 - d)}{2k} = \frac{d}{2k},$$

(9)

and

$$\pi^{(1)}(p^*_1) = M^* + \left(-\left(p^*_1\right)^2 + (p^*_1)^2 - (\mu - k)^2 - d^2\right) = M^* + \mu + \frac{d^2}{4k}.$$  

(10)

Because the seller incurs a cost of holding the property, his net expected payoff from this one-unit-time marketing period is

$$\pi = \pi^{(1)}(p^*_1) - g(1) = M^* + \mu + \frac{d^2}{4k} - g(1).$$  

Next, consider the case of a two-unit-time marketing period, i.e., the planned marketing duration is two units of time during which the seller can make another price offer $p_2$ to a second buyer if his first price offer $p_1$ was rejected by the first buyer. When the latter happens the seller realizes that his first offer price $p_1$ was set too high, and he rationally adjusts his second offer price $p_2$ downward based on his updated estimate of the property’s fundamental value. The possibility of offering his property to a second seller if the first selling attempt was unsuccessful allows a richer strategy set for the seller. Learning of the common fundamental value component occurs over the marketing period and serves as a basis for the offer price change.

With two offer prices $p_1$ and $p_2$ the seller’s problem becomes one of maximizing the following expected payoff:

$$\pi^{(2)}(p_1, p_2) = M^* + p_1 \cdot \Pr^{(1)}\{p_1 \leq f + d\} + \Pr^{(1)}\{p_1 > f + d\} \cdot \left\{ p_2 \cdot \Pr^{(2)}\{p_2 \leq f + d\} + \frac{1}{2}(\mu - k + (p_2 - d) \cdot \Pr^{(2)}\{p_2 > f + d\} \right\}.$$  

(11)
Here, \( \Pr^{(1)}\{p_1 \leq f + d\} \) and \( \Pr^{(1)}\{p_1 > f + d\} \) are the same as in the previous case and given by (5) and (6), and represent respectively the probabilities of the seller selling and not selling the property during the first unit of time with the price offer \( p_1 \). If the seller fails to sell his property in his first marketing attempt, he updates his prior distribution on \( f \), which is the uniform distribution \( f \sim U[\mu - k, \mu + k] \), to the posterior distribution \( f \sim U[\mu - k, p_1 - d] \) in a Bayesian fashion. Thus, the probabilities of the buyer accepting and rejecting the seller’s second price offer \( p_2 \) are respectively

\[
\Pr^{(2)}\{p_2 \leq f + d\} = \frac{(p_1 - d) - (p_2 - d)}{(p_1 - d) - (\mu - k)},
\]

(12)

and

\[
\Pr^{(2)}\{p_2 > f + d\} = \frac{(p_2 - d) - (\mu - k)}{(p_1 - d) - (\mu - k)}.
\]

(13)

Substituting (12) and (13) into the expression for \( \pi^{(2)}(p_1, p_2) \), after some algebra, the first-order conditions \( \frac{\partial \pi^{(2)}(p_1, p_2)}{\partial p_1} = 0 \) and \( \frac{\partial \pi^{(2)}(p_1, p_2)}{\partial p_2} = 0 \) lead to the following system of two linear equations with two unknowns:

\[
-2p_1 + p_2 + (\mu + k + d) = 0
\]

(14)

\[
p_1 - p_2 - d = 0
\]

(15)

whose solution gives the seller’s optimal offer price sequence: \( p_1^* = \mu + k \), \( p_2^* = \mu + k - d \). Thus the probability of trade \( \theta^{(2)} \) and the seller’s expected payoff \( \pi^{(2)}(p_1^*, p_2^*) \) from this two-unit-time marketing period can be computed as

\[
\theta^{(2)} = \Pr^{(1)}\{p_1^* \leq f + d\} + \Pr^{(1)}\{p_1^* > f + d\} \cdot \Pr^{(2)}\{p_2^* \leq f + d\} = \frac{d}{k},
\]

(16)
\[ \pi^{(2)}(p_1^*, p_2^*) = M^* + \mu + \frac{d^2}{2k}. \]

Again, due to the holding cost the seller’s net expected payoff in this case is

\[ \bar{\pi}^{(2)} = \pi^{(2)}(p_1^*, p_2^*) - g(2) = M^* + \mu + \frac{d^2}{2k} - g(2). \]

Using a similar line of reasoning and calculations, we can obtain the corresponding results for the three-unit-time marketing period as follows.

**Optimal sequence of offer prices:**  
\[ p_1^* = \mu + k, \quad p_2^* = \mu + k - d, \quad p_3^* = \mu + k - 2d \]

**Probability of trade:**  
\[ \theta^{(3)} = \frac{3d}{2k} \]

**Seller’s net expected payoff:**  
\[ \bar{\pi}^{(3)} = M^* + \mu + \frac{3d^2}{4k} - g(3) \]

Comparing heuristically the three sets of results obtained so far for the three different marketing lengths, a natural conjecture for the general \( n \)-unit-time marketing period is readily available. The conjecture turns out to be true and is stated in the following lemma.

**Lemma 1.** Suppose the seller markets his property for \( n \) units of time, \( n = 1, 2, ..., N \) where \( N \) is the seller’s maximum marketing horizon. During this marketing period the seller’s optimal sequence of offer prices are given by

\[ p_1^* = \mu + k, \quad p_2^* = \mu + k - d, \quad ..., \quad p_n^* = \mu + k - (n-1)d; \]  
the probability of trade from this marketing period is  
\[ \theta^{(n)} = \frac{nd}{2k}; \]  
and the seller’s gross and net expected payoffs are  
\[ \pi^{(n)} = M^* + \mu + \frac{nd^2}{4k} \quad \text{and} \quad \bar{\pi}^{(n)} = M^* + \mu + \frac{nd^2}{4k} - g(n) \] respectively.

Proof. See the Appendix.

The lemma indicates that the seller’s sequential search under buyer-sided private
information concerning the property’s fundamental value is characterized by an optimal sequence of monotonically decreasing offer prices. The seller receives incremental gains in his expected payoff as he extends the marketing length \( \pi^{(n)} \) is an increasing function of \( n \), but at the same time also incurs increasingly higher holding costs; this trade-off determines the seller’s optimal marketing length \( n^* = \arg \max \pi^{(n)} \) and the corresponding probability of trade \( \theta^{(n^*)} = \frac{n^*d}{2k} \). The latter is clearly a decreasing function of \( k \), the measure of market volatility described earlier.

We thus obtain the main result of this section, summarized in the following proposition.

**Proposition 2.** Suppose the buyer is informed about the fundamental value of the property for sale and the seller is not. The seller’s optimal search strategy yields a probability of trade that is decreasing with the market volatility. Consequently the liquidity of the property is negatively related to the market volatility, ceteris paribus.

4. Price Signaling with Seller-Sided Private Information

Having examined the case of an informed buyer trading with an uninformed seller and its implications for the volatility-liquidity relationship, we now turn to the case of an uninformed buyer trading with an informed seller. In the real estate market or any market in general, it is not clear, *a priori*, which side of the market will possess better information on a decision relevant variable than the other, if asymmetric information exists in the market at all. In this section we examine a setting in which the seller who privately learns the fundamental value of his property offers it to an
uninformed buyer for sale. The buyer’s belief about the fundamental value of the property is the uniform distribution \( f \sim U[\mu-k, \mu+k] \). Since the seller knows the fundamental value exactly, his price offer conveys information that the buyer may potentially use in making her purchase decisions. On the other hand, if the property is not sold to the first buyer, there is no learning taking place on the part of the seller because it is the seller himself who exclusively possesses private information about the property’s fundamental value. Consequently, when the seller faces a second buyer in his next selling attempt, an identical game with the same information structure is repeated between the seller and the second buyer. For this reason, it suffices to study the one-shot stage game for the purpose of investigating the volatility-liquidity relationship in the current setting.

We thus analyze a signaling game in which the seller signals his private information through his price offer, to which the buyer responds strategically given her updated information about the property’s fundamental value. We will characterize the perfect Bayesian equilibrium (PBE) of this signaling game. A strategy for the seller is a choice of offer price that is contingent on the property’s fundamental value privately observed by him, that is, the seller’s strategy is a (measurable) function that maps the set of all possible fundamental values, \([\mu-k, \mu+k]\), to the set of permissible offer prices. The buyer’s strategy is an acceptance rule determining which offer prices to accept and which ones to reject, when facing price offers from the seller. A PBE of this game consists of a pair of strategies, one each for the buyer and the seller, and a set of beliefs for the buyer concerning the true fundamental value \( f \).
upon seeing the seller’s price offers. Given the seller’s strategy and her own beliefs about \( f \), the buyer’s strategy must maximize her expected payoff. Similarly, given the buyer’s strategy the seller’s strategy must maximize his expected payoff. A final requirement of the PBE - the consistency condition - is that the buyer’s beliefs be derived from the seller’s equilibrium strategy whenever possible.

In signaling games like this with a continuum of types, typically there exist a plethora of equilibria which renders a meaningful comparative static analysis nearly impossible. The difficulty stems from the fact that, without further restrictions, the buyer’s acceptance set (i.e., the set of seller’s offer prices deemed acceptable by the buyer) can be quite arbitrary; for example, the buyer may choose to accept the seller’s offer prices \( p_1 \) and \( p_3 \) while rejecting offer price \( p_2 \) where \( p_1 < p_2 < p_3 \) - rather anomalous and counterintuitive behavior on the part of the buyer. To narrow down the set of possible PBEs we restrict the buyer’s acceptance set to be of a reservation price type. More precisely, if the buyer accepts the seller’s offer price \( p \) then she should also accept any offer price below \( p \); the least upper bound of the set of all acceptable offer prices thus forms the buyer’s reservation price. This is a more realistic scenario because, in practice, we do often observe real estate traders adopting such a trading strategy: accepting all offers below (or above) a threshold and rejecting those above (or below).

Let the buyer’s reservation price be denoted by \( r \) which we will subsequently treat as a parameter in the construction of the PBEs. Knowing the form of the buyer’s acceptance set, if the seller ever wants the buyer to accept his offer price in
equilibrium, he will be better off setting his offer price equal to the buyer’s reservation price (and not below it). On the other hand, if the seller’s equilibrium strategy calls for the buyer’s rejection of his offer price for a certain range of the property’s fundamental value, then the seller can set his offer price to equal \( \mu + k + d \) which will always be rejected by the buyer. Consequently we can, without loss of generality,\(^6\) assume that the seller chooses from two potential offer prices: \( r \) or \( \mu + k + d \), for each fundamental value of the property he observes.

The seller chooses the price offer \( r \) - intended to be accepted by the buyer - if and only if \( f \leq r \). We’ll search for those PBEs in which (a) \( r \leq \mu + k \), or (b) \( r > \mu + k \).

First consider case (a). In equilibrium, if the buyer sees the seller’s offer price \( r \), she will update her belief about the property’s fundamental value to the posterior distribution \( f \sim U[\mu - k, r] \), while if she sees the price offer \( \mu + k + d \), she will update her belief to \( f \sim U[r, \mu + k] \). Rationality on the part of the buyer implies that she will accept the seller’s offer price \( r \) if and only if \( \frac{1}{2}[(\mu - k) + r] + d - r \geq 0 \), or, equivalently \( r \leq \mu - k + 2d \). This latter condition means that if the reservation price \( r \) is to be an equilibrium strategy of the buyer, it has to lie in the interval \( [\mu - k, \mu - k + 2d] \). Note that since \( k > d \) and hence \( \mu - k + 2d \leq \mu + k \), \( r \leq \mu - k + 2d \) is a more restrictive condition than \( r \leq \mu + k \), the original condition in (a). Conversely, it can be easily checked that for any \( r \in [\mu - k, \mu - k + 2d] \), there exists a PBE in which the buyer’s reservation price is \( r \).

For case (b), a necessary condition for the buyer’s reservation price \( r \) to be an

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\( ^{6} \) The phrase “without loss of generality” here means such a strategy of the seller is outcome-equivalent to any other equilibrium strategy of the seller, given the buyer’s equilibrium strategy.
equilibrium strategy is the same as in case (a): $r \leq \mu - k + 2d$. But this contradicts $r > \mu + k$, because $k > d$ as assumed. Therefore, there exists no PBE in which $r > \mu + k$.

Summing up, the set of all PBEs of the signaling game are indexed by the buyer’s reservation price $r \in [\mu - k, \mu - k + 2d]$. For any $r \in [\mu - k, \mu - k + 2d]$, the PBE can be described as follows: The buyer sets her reservation price at $r$; the seller offers his property to the buyer at price $r$ if he observes a fundamental value $f \leq r$ and offers the price $\mu + k + d$ if $f > r$; the buyer updates her belief about the property’s fundamental value to $f \sim U[\mu - k, r]$ upon seeing the seller’s price offer $r$ and updates it to $f \sim U[r, \mu + k]$ upon seeing price offer $\mu + k + d$; and the buyer accepts the seller’s price offer $r$ and rejects his price offer $\mu + k + d$.

In order to derive meaningful comparative static results we will restrict our attention to the PBE which maximizes the seller’s expected payoff. Clearly, this PBE is the one with $r = \mu - k + 2d$. Also clear is the fact that this PBE maximizes the probability of trade as well, with the maximum probability of trade given by

$$\Pr\{\mu - k \leq f \leq \mu - k + 2d\} = \frac{d}{k}$$

because the seller will set his offer price equal to $r = \mu - k + 2d$ (and not $\mu + k + d$) if and only if $\mu - k \leq f \leq \mu - k + 2d$ as argued before. Once again we find an inverse relationship between real estate liquidity and market volatility: the probability of trade, $\frac{d}{k}$, decreases with the volatility parameter, $k$.

The following proposition summarizes the findings of this section.

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7 More precisely, the seller’s payoff from this PBE first-order stochastically dominates those from other PBEs.
Proposition 3. Suppose the seller is informed about the fundamental value of the property for sale and the buyer is not. In the perfect Bayesian equilibrium (PBE) of the signaling game that maximizes the seller’s expected payoff (which happens to also maximize the probability of trade), the probability of trade is decreasing with the market volatility. Consequently, the liquidity of the property is negatively related to the market volatility, ceteris paribus.

There are two potential justifications for the choice of the particular PBE as the likely outcome of the signaling game. First, as shown in Section 2 for the case of symmetric information, the seller, by virtue of his first-mover advantage, may have the ability to effect an outcome of the bargaining game that accrues to him all of the benefits from trade. A similar case can be made in the current situation of asymmetric information: His “market power” augmented with an informational advantage over his counter-party enables the seller to choose an equilibrium that is most advantageous to him. Another justification draws on the work of Yavas (1995), who shows that real estate brokers can play a welfare improving and equilibrium selecting role by coordinating the activities of the buyer and seller and reducing the number of equilibria in the market. Note that the particular PBE selected in our signaling game (i.e., the one with \( r = \mu - k + 2d \)) achieves (constrained) social efficiency because it maximizes the probability of trade and thus maximizes the total expected surplus among all PBEs. The second justification, therefore, is to allow an implicit role for

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8 The general and more difficult problem of equilibrium refinement is beyond the scope of the current paper.

9 The total surplus is \( d \) as long as trade occurs; the lost welfare results from the possibility of no trade for those \( f \) within the range \( (r, \mu - k + 2d) \) if the equilibrium has \( r < \mu - k + 2d \). However, this PBE does not achieve the first-best efficiency which would require trade occur for all \( f \in [\mu - k, \mu + k] \), and hence is a second-best (or constrained) efficient outcome given asymmetric information.
brokers in our model who may then serve to bring the trading parties onto an equilibrium that maximizes the social surplus. To the extent that real estate brokers are compensated chiefly on successful deals, it is clearly in their interest to improve the probability of trade between the buyer and the seller they serve.

The seller-offer, ultimatum bargaining game considered here and in Krainer (2001) is certainly not the only approach to modeling the bargaining behavior in the real estate market.\(^{10}\) Several authors have considered alternative bargaining procedures and the associated information structures. For example, Yavas (1992) and Yavas and Yang (1995) study a “reduced-form” bargaining game where the outcome is determined by the players’ relative bargaining powers and is related to the solutions of Nash (1950) and Rubinstein (1982). In both papers there is no private information at the bargaining stage. Arnold (1999) describes an alternating-offer bargaining game in which both the buyer and the seller incur a cost of delay in the form of discounted returns. He characterizes the subgame perfect equilibrium (SPE) of the game under the assumption of complete information. As acknowledged by Yavas (1992) and Arnold (1999), modeling the bargaining behavior under incomplete information for the real estate market might be both more realistic and challenging especially in view of the existence of multiple equilibria. The model and equilibrium selection presented in this section may be viewed as a first step toward a more complete understanding of an issue that has practical relevance for the modeling of real estate transactions.

\(^{10}\) The seller-offer, ultimatum bargaining game is also employed in Read (1988).

The primary setting of the current paper is one of common value, that is, there is a common-value component in both the buyer’s and the seller’s valuation of the property for sale, albeit unknown to one or both of the parties at the time of trading. By contrast, many papers in the existing literature have considered settings of private value in the context of marketing real estate. One of the most prominent among them is Haurin’s (1988) seminal work on the relationship between a property’s marketing time and its degree of atypicality. He argues that houses with greater atypicality tend to attract offers with greater variance. Because a profit-maximizing seller will wish to increase the property’s marketing exposure in such circumstances in an attempt to fetch a higher selling price, atypical houses sit on the market longer than standard houses, which translates into lower liquidity for atypical houses. The author thus hypothesizes an inverse relationship between a house’s atypicality and its liquidity, which is then borne out theoretically and empirically in the paper.

Note that the atypicality of a house in Haurin’s (1988) model refers to those of the house’s characteristics that are typically unobservable by the seller and idiosyncratic to the potential buyers; for example, the aesthetics of the house which by nature are buyer-specific. Importantly, the seller’s own valuation of his house is independent of the value of the house’s idiosyncratic characteristics to the potential buyers. The fact that a particular buyer places an extremely high or extremely low value on those characteristics, for example, does not in any way alter the house’s value to the seller.

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himself. In this sense, the setting in Haurin’s (1988) paper is one of private value. Because the idiosyncratic value of the house is known only to the buyer herself, the setting is also one of asymmetric information. Note also that the atypicality of a house in Haurin’s (1988) paper serves as an indirect measure of the extent to which the buyers’ offer prices may vary during the marketing period.\footnote{Deng, Gabriel, Nishimura and Zheng (2012) measure the house price dispersion more directly.} Atypicality thus captures the volatility of a house’s value among the pool of potential buyers. Under this interpretation, what Haurin (1988) has established is essentially an inverse relationship between a property’s value volatility and its liquidity in a setting characterized by private value and asymmetric information.

In this section we extend the main theme of the current paper, i.e. the examination of the volatility-liquidity relationship, to the case of private value. We revisit Haurin’s (1988) problem by studying a variation on his original model. Our purpose is to provide a “robustness check” on the main findings of this celebrated work using an alternative model specification.

In Haurin’s (1988) original model, the seller, in the face of uncertainty about the potential buyers’ offer prices, conducts a sequential search in which he trades off the potential benefit from waiting for another (better) offer against the cost of waiting, in the form of an optimal stopping rule. The seller’s role is one of waiting passively for the potential buyers’ price offers, whose probability distribution is known to the seller. In our variant of the model, we consider a one-shot ultimatum bargaining game where an uninformed seller makes a take-it-or-leave-it price offer to an informed buyer,
similar to the one studied in Section 4. The value of the house to the buyer and the
seller remains the same as before: $v^b = f + d$ for the buyer and $v^s = f'$ for the seller
with $d > 0$. Unlike in the previous sections, though, we now assume the common
value component, $f$, is known to both parties but the idiosyncratic value component, $d$,
is observed only by the buyer - an assumption akin to that in Haurin (1988).

Another departure from Haurin’s model (1988) concerns the way in which the
volatility of the property’s idiosyncratic value is modeled. In Haurin (1988) and
Haurin, Haurin, Nadauld and Sanders (2010) the buyer’s offer price is assumed to be
uniformly or normally distributed. Deng, Gabriel, Nishimura and Zheng (2012) also
use normal distribution to model the dispersion of the buyers’ offer prices. In more
general works on real estate trading, probability distributions of a particular kind are
usually assumed for the relevant property values or prices. All these are examples of
what is referred to as the *cumulative distribution function (CDF) approach* to
modeling risk and volatility (Meyer and Ormiston, 1989). Under this approach, for the
particular cases of uniform and normal distributions, one distribution is considered
more risky or more volatile than another if the two distributions have the same mean
but the former has a larger variance. Indeed, this is the most familiar and widely used
concept of risk and volatility in the profession. In our modified model we employ
another approach, the *transformation approach* (Meyer and Ormiston, 1989), to
model the volatility of the property value, as detailed below.

To make comparisons on the volatility of the house’s idiosyncratic value $d$, under
the transformation approach, $d$ is assumed to be indexed by a scalar $k > 0$: $d = d_k$. 

28
Specifically, \( d \equiv d_k \) is constructed from a base random variable \( d_i \) via a linear transformation: 
\[
d_k = \mu + k(d_i - \mu),
\]
where \( \mu \) is the mean of \( d_i \) whose support is \([d^L, d^H]\) with \( d^H > d^L > 0 \), whose c.d.f. is \( H_i(\cdot) \), and whose variance is \( \sigma^2 \). The requirement \( d_k > 0 \) puts a natural restriction on how large the parameter \( k \) can be: 
\[
\mu + k(d^L - \mu) > 0.
\]
For each \( k \) in the appropriate range, \( d_k \) has a mean equal to \( \mu \) and variance equal to \( k^2\sigma^2 \). Thus, as in the CDF approach, \( d_{k_2} \) is considered more volatile than \( d_{k_1} \) if \( k_2 > k_1 \). Note that, unlike in the CDF approach, no particular distributional form (e.g. uniform or normal) is specified for the base random variable \( d_i \) and hence, any \( d_k \).

The one-shot ultimatum bargaining game has the seller making a take-it-or-leave-it offer to a buyer whose valuation of the house’s idiosyncratic characteristics is unknown to the seller.\(^{13}\) The seller’s total valuation of the house is \( v^i = f \) and the buyer’s total valuation is \( v^b = f + d_k \). The fundamental value component \( f \) is common knowledge among the traders, while the idiosyncratic value component \( d_k \) is known only to the buyer.

**Proposition 4.** *In the bargaining game between the seller and the buyer described above, the probability of trade \( \theta \) decreases with \( k \). Thus the property’s liquidity is negatively correlated with the volatility of its idiosyncratic value, ceteris paribus.*

Proof. See the Appendix.

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\(^{13}\) Since we interpret \( d \) as the value to the potential buyers of the house’s idiosyncratic characteristics, it is independent across the potential buyers as in Haurin (1988). Thus, for example, if the seller does not sell his house to the first buyer, he learns nothing from this fact about the value of the house’s idiosyncratic characteristics to a second buyer; his posterior distribution of \( d \) remains the same as the prior distribution. Hence it suffices to consider a one-shot stage game in our investigation of the volatility-liquidity relationship, for the same reason as in Section 4.
As stated earlier our main purpose in this section is to provide some theoretical corroboration for the main findings of Haurin (1988) in the framework of the current paper. To this end, two major modifications are made to the specification of the original model. The first concerns the relative role played by the traders. In Haurin’s (1988) paper the potential buyers make all the price offers to the seller based on their idiosyncratic valuation of the house’s “atypical” characteristics. In our model, by contrast and in line with the framework of the current paper, the seller is assumed to make the price offer in an ultimate bargaining game, with the knowledge that he is facing a heterogeneous pool of potential buyers.

The second modification has to do with the way volatility is modeled. The theory of risk tells us that the most general concept of a change in risk/volatility is that of second-order stochastic dominance (SSD) or, equivalently, mean-preserving spread (MPS) (Eeckhoudt, Gollier and Schlesinger, 2005). The changes in volatility defined both in the CDF approach and the transformation approach can be shown to be special cases of those defined in the more general SSD/MPS approach, using the single-crossing property of SSD/MPS (Wolfstetter, 1999). The advantage of using the more restrictive volatility concept in the CDF or transformation approach is that it allows for comparative static results to be derived in cases where the SSD/MPS approach fails (Meyer and Ormiston, 1989). Thus, by casting Haurin’s (1988) original model under the new light of an alternative volatility concept and reproducing essentially the same result (i.e., the negative volatility-liquidity relationship), our analysis in this section lends support to the validity of Haurin’s (1988) main findings.
6. Conclusion

The issue of what determines the real estate liquidity is a topic of perennial interest to the real estate researchers. We provide the first formal analysis on how market volatility affects liquidity in the real estate market. The investigation is carried out in a setting characterized by both common and private value as well as asymmetric information, and different information structures are examined in the context of a seller-offer, ultimatum bargaining game. Under symmetric information, i.e., when the buyer and the seller are either both informed or both uninformed, we find that changes in the market volatility have no effect on its liquidity. Under asymmetric information, in contrast, there exists a negative relationship between the market volatility and its liquidity. These findings can potentially explain phenomena that occurred during the 1997 Asian Financial Crisis when the volume of property transactions were significantly reduced in the real estate market as investors became increasingly uncertain about the future market trend.

We contribute to the literature by studying how the market volatility (the second moment) affects real estate liquidity. Our work complements that of Krainer (2001) who identifies a level effect of the market-wide value component (the first moment) on market liquidity. While Krainer (2001) obtains a positive state-liquidity relationship in his paper, we demonstrate a negative volatility-liquidity relationship for the cases of asymmetric information. As an extension and by-product of our analysis, we also reaffirm the main findings of Haurin’s (1988) classic work under an
alternative model setting.

As in Krainer (2001), we adopt the probability of sale as a liquidity measure throughout our analysis, which is less frequently used in the literature than the alternative measure of time-on-market (TOM). To the extent that transaction volume in a given time period is a more direct and measurable indicator of the market conditions, the probability of sale, arguably more readily linked to the transaction volume than TOM, might serve as a measure of the real estate liquidity no less desirable than TOM. Indeed, the two alternatives can be shown to be equivalent under stable market conditions.\textsuperscript{14} The question arises as to which constitutes the better measure of market liquidity when the stability in market conditions cannot be assumed (Anglin, 2006; Johnson, Benefield and Wiley, 2007). To our knowledge this question has not yet been addressed in the literature. We leave an exploration of this theoretical issue for future research.

\textsuperscript{14} Rutherford, Springer and Yavas (2005) and Deng, Gabriel, Nishimura and Zheng (2012) assume the equivalence between the two alternative measures.
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Appendix

(A) Proof of Lemma 1

We prove the lemma by induction. In Section 3 of the main text we have already shown that the lemma is true for the base cases of \( n = 1, 2, 3 \). Suppose the lemma is true for \( n \). We want to show the lemmas is true for \( n + 1 \).

The induction hypothesis that the lemma is true for \( n \) means the following:

*For a marketing period of length \( n \), the seller’s optimal sequence of offer prices are given by \( p_1 = \mu + k \), \( p_2 = \mu + k - d \), \( \ldots \), \( p_n = \mu + k - (n - 1)d \); the probability of trade from this marketing period is \( \theta^{(n)} = \frac{nd}{2k} \); and the seller’s gross and net expected payoffs are \( \pi^{(n)} = M^s + \mu + \frac{nd^2}{4k} \) and \( \bar{\pi}^{(n)} = M^s + \mu + \frac{nd^2}{4k} - g(n) \) respectively.*

Note that all the results in the induction hypothesis are assumed under the condition that the prior distribution of the fundamental value is \( U[\mu - k, \mu + k] \).

Now, consider a marketing period of length \( n + 1 \). Suppose the seller’s first offer price is \( p_1 \) which is rejected by the first buyer. After the rejection of the first offer by the first buyer, by the stationarity of the seller’s sequential search, the seller now faces an identical, new search problem of marketing length \( n \) and the new prior distribution of the fundamental value \( U[\mu - k, p_1 - d] \), which is seller’s posterior distribution following the rejection of the first offer price \( p_1 \). To conform to the form of the condition used in the induction hypothesis, rewrite \( U[\mu - k, p_1 - d] \) as \( U[\mu' - k', \mu' + k'] \) where

\[
\mu' = \frac{(p_1 - d) + (\mu - k)}{2},
\]
\( k' = \frac{(p_1 - d) - (\mu - k)}{2} \).

The induction hypothesis implies that the optimal sequence of offer prices starting from the second offer should be \( p'_2 = \mu' + k' \), \( p'_i = \mu' + k' - d \), ..., \( p'_{n+1} = \mu' + k' - (n-1)d \), and the seller’s gross expected payoff from the new search should be \( \tilde{\pi}^{(n)} = M' + \mu' + \frac{nd^2}{4k'} \). Note that \( \tilde{\pi}^{(n)} \) is a function of \( p_1 \) since both \( \mu' \) and \( k' \) are a function of \( p_1 \).

Thus, the seller’s problem is to choose \( p_1 \) to maximize the following expected payoff, given the optimality of the search after the rejection of the first offer:

\[
\tilde{\pi}^{(n+1)} = (M' + p_1) \cdot \Pr\{p_1 \leq f + d\} + \tilde{\pi}^{(n)} \Pr\{p_1 > f + d\} = M' + p_1 \cdot \left( \frac{\mu + k - (p_1 - d)}{2k} + \frac{(p_1 - d) - (\mu - k)}{2k} \right) \left[ \mu' + \frac{nd^2}{4k'} \right].
\]

The first-order condition \( \frac{\partial \tilde{\pi}^{(n+1)}}{\partial p_1} = 0 \), after some tedious algebra, yields \( p_1 = \mu + k \).

We thus get

\[
\mu' = \frac{(p_1 - d) + (\mu - k)}{2} = \frac{2\mu - d}{2},
\]

\[
k' = \frac{(p_1 - d) - (\mu - k)}{2} = \frac{2k - d}{2}.
\]

Substituting the above expressions of \( \mu' \) and \( k' \) into \( p'_2 = \mu' + k' \), \( p'_3 = \mu' + k' - d \), ..., \( p'_{n+1} = \mu' + k' - (n-1)d \), we get the seller’s optimal sequence of offer prices for the marketing length \( n+1 \):

\[
p'_1 = p_1 = \mu + k, \quad p'_2 = \mu + k - d, \quad p'_3 = \mu + k - 2d, \ldots, \quad p'_{n+1} = \mu + k - nd.
\]

With this information we can easily compute the other items under consideration, which turn out to agree with the results of the lemma for the marketing length \( n+1 \).

The proof is thus complete.
(B) Proof of Proposition 4

Let \( h_1(\cdot) \) denote the density function of the c.d.f. \( H_1(\cdot) \). Suppose the seller makes a price offer \( p \) to the buyer. Since the buyer is informed about the fundamental value of the property, she will accept the seller’s offer if and only if \( p \leq f + d_k \). Thus, the seller’s expected payoff if he offers price \( p \) to the buyer is

\[
\pi^*(p) = M' + f \cdot \Pr\{p > f + d_k\} + p \cdot \Pr\{p \leq f + d_k\}
\]

\[
= M' + f \cdot H_1\left(\mu + \frac{p - f - \mu}{k}\right) + p \cdot \left[1 - H_1\left(\mu + \frac{p - f - \mu}{k}\right)\right]
\]

The first-order condition \( \frac{\partial \pi^*(p)}{\partial p} = 0 \) gives

\[1 - H_1\left(\mu + \frac{p - f - \mu}{k}\right) = \frac{p - f}{k} \cdot h_1\left(\mu + \frac{p - f - \mu}{k}\right).\]  \hspace{1cm} (*)

The second-order condition, which we assume to hold, is given by

\[-2h_1\left(\mu + \frac{p - f - \mu}{k}\right) - \frac{p - f}{k} \cdot h_1\left(\mu + \frac{p - f - \mu}{k}\right) < 0.\] \hspace{1cm} (**)

Equation (*) implicitly defines \( p \) as a function of \( k \), which we henceforth denote as \( p(k) \). Differentiating both sides of (*) with respect to \( k \) and treating \( p \) as a function of \( k \), we get

\[
\left[p'(k) - \frac{p - f - \mu}{k}\right] \left[2h_1\left(\mu + \frac{p - f - \mu}{k}\right) + \frac{p - f}{k} \cdot h_1\left(\mu + \frac{p - f - \mu}{k}\right)\right] = h_1\left(\mu + \frac{p - f - \mu}{k}\right) \cdot \mu
\]

The expression in the second pair of brackets on the left-hand side of the above equation is positive by (**). Since the right-hand side of the above equation is also positive, we must have

\[p'(k) - \frac{p - f - \mu}{k} > 0.\] \hspace{1cm} (***)
Now, consider the probability of trade \( \theta(k) = 1 - H_h \left( \mu + \frac{p - f - \mu}{k} \right) \). We have

\[
\frac{\partial \theta(k)}{\partial k} = -\frac{1}{k} \cdot h \left( \mu + \frac{p - f - \mu}{k} \right) \cdot \left[ p'(k) - \frac{p - f - \mu}{k} \right] < 0
\]

by (**). Thus we have shown that the probability of trade \( \theta \) decreases with \( k \).